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The empirical correlation for forced convection heat transfer of homogeneous hybrid nanofluid $\text{TiO}_2/\text{Al}_2\text{O}_3$ in a non-uniform heat flux test section

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HIGHLIGHTS

- Analysis of forced heat transfer in an annulus using a homogeneous hybrid nanofluid with a cosine heat flux.
- Experimental design to study the effects of nanoparticle volume fraction and Reynolds number on heat transfer.
- Creation of correlations to predict the heat transfer behavior of the nanofluid.
- Potential for using nanofluids as an emergency core coolant.

ABSTRACT

In this research, an empirical correlation is derived to determine the Nusselt number for nanofluid forced convection flow in a vertical annulus with non-uniform heat flux. The experiments investigate the heat transfer of a homogeneous combined nanofluid composed of TiO_2 and Al_2O_3 dispersed in water. Compared to other conventional fluids, nanofluids exhibit better heat transfer performance due to higher conductivity and reducing the thermal boundary layer. Nanofluids have the potential to function in an emergency core cooling system. A 25-bar test loop with a vertical test section generating non-uniform heat flux is used for the experiment. The tests are conducted at multiple Reynolds numbers and volumetric concentrations of the nanofluid. A new correlation for nanofluid heat performance is developed based on dimensional analysis and multiple linear regression analysis. The Nusselt number derived from the developed correlation shows high accuracy, with $R^2 = 0.92$, indicating better performance compared to existing correlations.

KEYWORDS

Hybrid nanofluid
 $\text{Al}_2\text{O}_3\text{-TiO}_2$
 Empirical correlation
 Forced convection

HISTORY

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Nomenclature

| | |
|--------------|---|
| C_p | specific heat ($\text{J.kg}^{-1}.\text{K}^{-1}$) |
| h | heat transfer coefficient ($\text{W.m}^{-2}.\text{K}^{-1}$) |
| k | thermal conductivity ($\text{W.m}^{-1}.\text{K}^{-1}$) |
| l | length of heating rod (m) |
| Nu | Nusselt number |
| P | pressure (bar) |
| Q | total heat generation (W) |
| Pr | Prantle number |
| Re | Reynolds number |
| R^2 | coefficient of determination |
| T | temperature ($^{\circ}\text{C}$) |
| U | uncertainty |
| V | velocity (m.s^{-1}) |
| μ | viscosity ($\text{kg.s}^{-1}.\text{m}^{-1}$) |
| ρ | density (kg.m^{-3}) |
| ϕ | volumetric concentration of nanofluid |
| ΔT_m | logarithmic mean temperature difference |

Subscripts

| | |
|------|--------------|
| bf | base fluid |
| nf | nanofluid |
| np | nanoparticle |

1 Introduction

A significant challenge in improving thermal performance is the low heat transfer of typical working fluids (Borhani et al., 2023; Sajid and Ali, 2019). Nanofluids, which consist of nanoparticles (particles under 100 nm) dispersed in a base fluid, show greater heat transfer capabilities compared to typical fluids (Abbassi et al., 2014; Talebi et al., 2022). The heat transfer capabilities in nanofluids make them a promising option for application in emergency core cooling systems (ECCS) in nuclear reactors (Chinchole et al., 2019). The innovative study of Choi et al. (Choi

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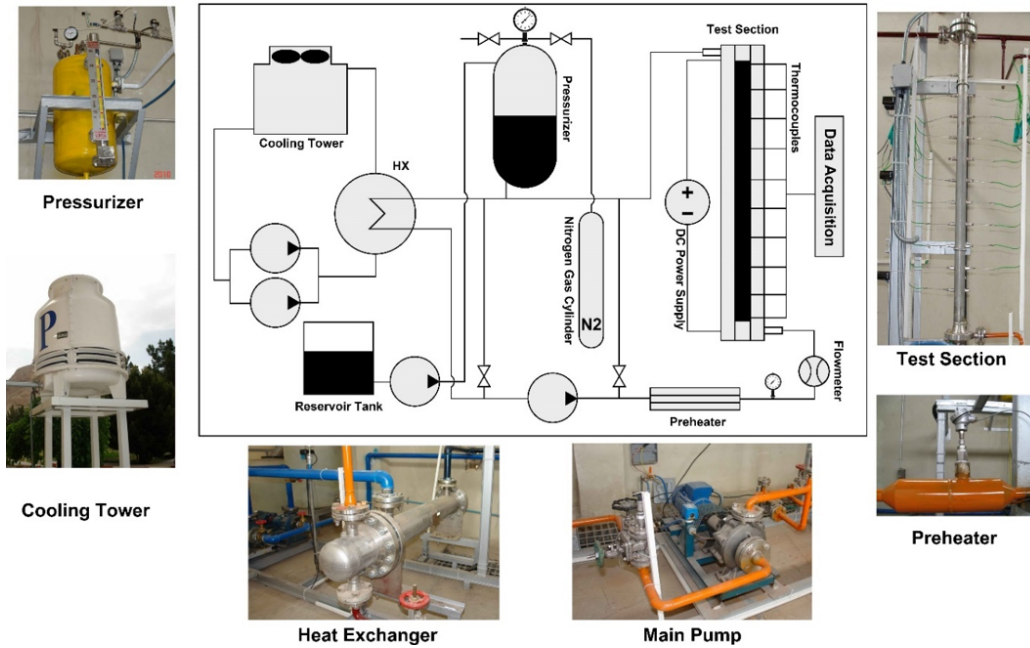


Figure 1: Schematic and pictures of the experimental setup.

and Eastman, 1995) recommended nanofluids as working fluids in thermal systems. Recent studies have demonstrated the utilization of combined nanofluids, instead of single nanofluid applications (Vallejo et al., 2022).

The better thermal characteristics of nanofluids are attributed to different mechanisms, such as the rise in thermal conductivity, the Brownian motion of particles, and reducing the thermal boundary layer (Wang et al., 2021; Borhani et al., 2024). Several studies have examined the specific heat capacity, thermal conductivity, viscosity, and heat transfer of nanofluids (Alklaibi et al., 2022; Ataei et al., 2020; Chebbi, 2015; Esfe and Saedodin, 2014; Huang et al., 2016; Hwang et al., 2007). Al_2O_3 and TiO_2 nanoparticles are widely used in nanofluid research. Pak et al. (Pak and Cho, 1998) focused on the heat transfer behaviors of dispersed fluids containing submicron metallic oxide particles suspended in water. He et al. (He et al., 2007) conducted an experimental study to measure the heat transfer and pressure drop of water- TiO_2 nanofluids in both laminar and turbulent flows. Duangthongsuk et al. (Duangthongsuk and Wongwises, 2009) observed the impact of nanoparticles of TiO_2 in a thermal exchanger and discovered that adding a small amount of nanoparticles increases pump work while also improving heat transfer efficiency.

Various methods, including dimensional analysis, multivariate regression, symbolic regression with genetic algorithms, or neural networks (Borhani et al., 2023), have been utilized to establish correlations for determining the thermal-hydraulic properties of nanofluids (Sánchez-Escalona et al., 2022; Xuan and Roetzel, 2000).

In this study, a pressurized test facility with cosine heat flux was employed. An ultrasonic stirrer was used to stabilize homogeneous solutions of Al_2O_3 and TiO_2 nanoparticles (same volume) in water. The experiments were conducted at multiple Reynolds numbers and nanofluids con-

centrations. An empirical correlation for Nusselt numbers was developed to compare with other correlation equations and experimental results. Dimensional analysis and multiple linear regression were utilized to obtain the Nusselt number correlation. This research can be applied particularly to heat exchangers and ECCS system design.

2 Methodology and data analysis

2.1 Experimental model

The test loop in this study is a thermohydraulic loop with a maximum pressure of 25 bars. This test facility includes different components, such as pipes, a heat exchanger, pumps, a cooling tower, a test section, and other measurement and control instruments. Figure 1 presents a schematic of the test facility.

The test section consists of an annulus with an inner rod that produces a non-uniform heat flux of 1 kW (cosine shape). The outer diameter of the heating rod is 33 mm, the inner diameter of the casing is 55 mm, and the length of the inner heating rod is 1 meter. The rod's temperature is monitored using 22 type K thermocouples fixed to its surface, with the thermocouples linked to the data collection system.

For the preparation of the nanofluid in this experiment, an ultrasonic stirrer was employed. Initially, a homogeneous mixture of TiO_2 and Al_2O_3 nanoparticles (same volume) was added to water. To avoid the aggregation or clustering of nanoparticles, the ultrasonic stirrer was applied to the solution for 4 hours. The experiments were performed at an inlet fluid temperature of 25 °C throughout the tests. Nusselt numbers were derived from the measured parameters under a variety of operational conditions. A comprehensive summary of these conditions is available in Table 1.

Table 1: Information on the experimental conditions.

| Parameter | Value |
|---------------------------|------------------------------|
| Volumetric concentrations | 0.25%, 0.5%, 0.75%, 1%, 1.5% |
| Reynolds numbers | 1238, 1857, 2683, 3096, 4128 |
| Inlet Pressure | 5 bars |
| Inlet Temperature | 25 °C |
| Heat generation | 1 kW |

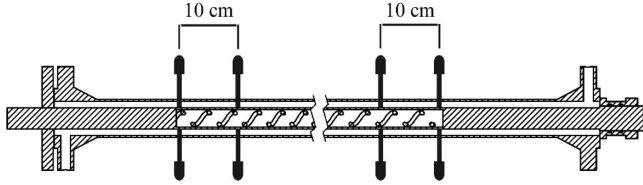


Figure 2: The locations of thermocouples in the test section (the test section is vertical).

Thermocouple locations are shown in Fig. 2. The distance between each thermocouple is 10 cm. The thermocouples were tightened and securely fixed in the test section to ensure precise temperature measurements.

Before conducting the experiment, precise calibration was performed on the measurement instruments, comprising the flowmeter and thermocouples. The calibration of the thermocouples was performed utilizing the water bath technique, a method known for its reliability and precision in temperature measurement. In addition to this, a calibrated reference flowmeter was employed to verify the accuracy of the primary flowmeter used in the system.

2.2 Data analysis

The nanofluid exhibits Newtonian behavior at low concentrations (Rizwan et al., 2022). The volumetric concentration was used for the estimation of density and thermal conductivity. The viscosity of the nanofluid (μ_{nf}) and thermal conductivity (k_{nf}) were determined using correlations. The viscosity of the nanofluid was calculated based on the correlation equation proposed by Wang et al. (Wang et al., 1999). Additionally, the equation developed by Timofeeva et al. (Timofeeva et al., 2007) was used to determine the nanofluid's thermal conductivity. Equations (1) and (2) represent the viscosity and thermal conductivity of the nanofluid, respectively. The volume concentration (ϕ), nanofluid (nf), and base fluid (bf) are used to represent the variables in the following equations.

$$\mu_{nf} = \mu_b(1 + 7.3 \phi + 123 \phi^2) \quad (1)$$

$$k_{nf} = k_b(1 + 3 \phi) \quad (2)$$

The heat transfer coefficient (h) is computed using the following equations, based on the data obtained from thermocouples, the geometric parameters of the test section, and the heat generation within the test section (Wang et al., 2020).

$$h = \frac{Q}{(\pi D_{rod} l) \Delta T_m} \quad (3)$$

$$\Delta T_m = \frac{\Delta T_{max} - \Delta T_{min}}{\ln\left(\frac{\Delta T_{max}}{\Delta T_{min}}\right)} \quad (4)$$

where Q is the total heat generation in the test section, l is the heat rod's length and D_{rod} is its diameter. respectively, and ΔT_m is the logarithmic mean temperature difference. This temperature difference is obtained using the inner and outer fluid temperatures and the rod surface temperature. ΔT_{max} and ΔT_{min} are calculated as follows:

$$\Delta T_{max} = \frac{1}{5} \sum_{i=1}^5 T_{rod\ i} - T_{nf\ in} \quad (5)$$

$$\Delta T_{min} = \frac{1}{5} \sum_{i=6}^{10} T_{rod\ i} - T_{nf\ out} \quad (6)$$

The values for the Nusselt number and Prandtl number are obtained using Eqs. (7) and (8). Heat transfer is computed as the basis of the energy equation, bulk temperature, and rod surface temperature.

$$\bar{N}u = \frac{hD}{k_{nf}} \quad (7)$$

$$Pr = \frac{\mu_{nf} C_{nf}}{k_{nf}} \quad (8)$$

In this experiment, an uncertainty analysis was performed for each parameter, and calculated based on sensor accuracy and calibration error for various parameters. Equation (9) shows the law of uncertainty propagation, which calculates the overall uncertainty based on the uncertainties of the individual contributing variables.

$$U_f = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} U_{x_i}\right)^2} \quad (9)$$

The uncertainty for the Nusselt number is calculated based on Fig. 3. In this experiment, the uncertainty of the Nusselt number is approximately 4.3%.

In the initial design of the experiment, test loop pressure was considered. However, the effects of this parameter were found to be negligible. The most significant factors influencing the Nusselt number were identified as the mass flow rate and the concentration of the nanofluid. Each experiment was also repeated three times, and the average value was chosen for the calculations.

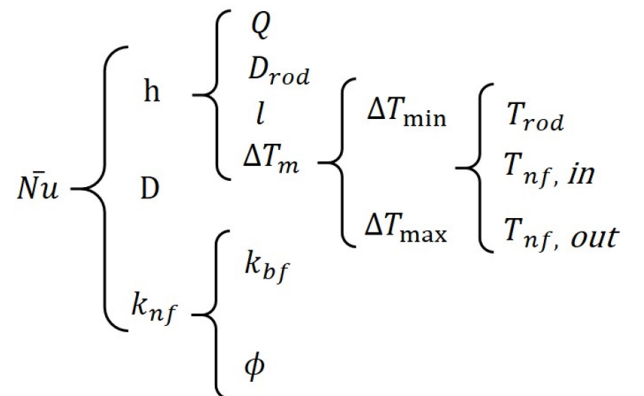


Figure 3: Uncertainty propagation to calculate the uncertainty of Nusselt number.

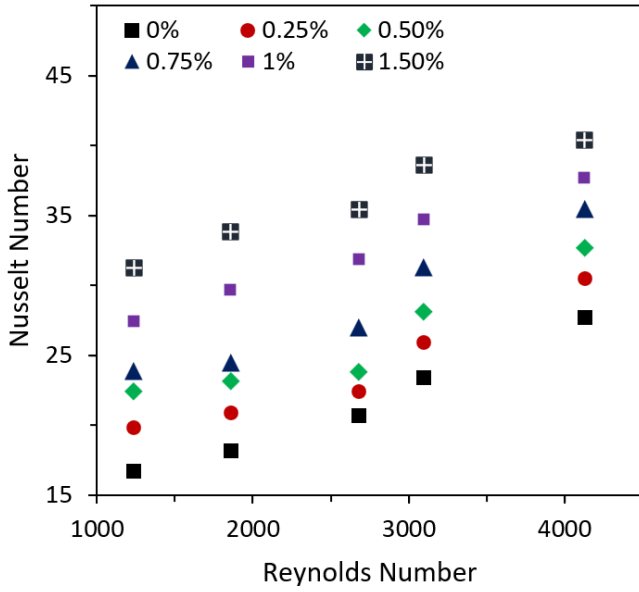


Figure 4: Experimental Nusselt numbers at diverse Reynolds numbers and concentrations.

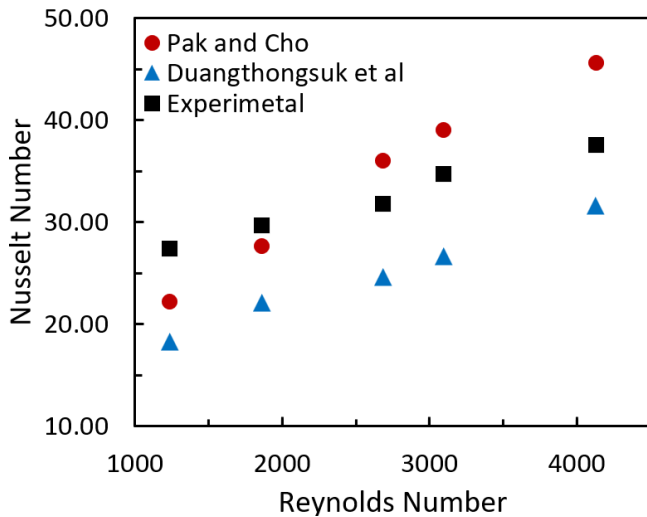


Figure 5: Comparison of the correlations and experimental Nusselt numbers for 1% nanofluid.

2.3 Correlation development

Correlation can be achieved through the use of dimensional analysis, which simplifies the problem and expresses it in a more straightforward physical relationship. A dimensionless group is needed to calculate heat transfer for different states. The equation can be written as follows:

$$h = f(\rho \mu c_p K V D \phi) \quad (10)$$

For the nanofluid, the thermal properties are calculated at bulk temperature. The equation is modified using Dimensional Analysis and Buckingham’s Pi method into the following format:

$$Nu = f(Re Pr \phi) \quad (11)$$

For the nanofluid, the thermal properties are calculated at bulk temperature. The equation is modified using

dimensional analysis and Buckingham’s Pi method into the following format:

$$Nu = a Re^b Pr^c (1 + \phi)^d \quad (12)$$

The coefficient of determination, referred to as R^2 , shows the variation in the dependent variable that can be anticipated based on the independent variables.

$$R^2 = 1 - \frac{\sum (y_i - f_i)^2}{\sum (y_i - \bar{y})^2} \quad (13)$$

In the R^2 formula, y_i represents the observed values, f_i denotes the predicted values, and \bar{y} indicates the mean of the observed values. The coefficients of a , b , c , and d were calculated using the multiple linear regression analysis method based on the data obtained from the experiments conducted with Python. This method is used to investigate how a dependent variable relates to two or more independent variables. To find the coefficients of a power relationship using linear regression, the data was transformed using logarithms, which made the format of the formula linear. This is shown in Eq. (14):

$$\ln(Nu) = \ln(a) + b \ln(Re) + c \ln(Pr) + d \ln(1 + \phi) \quad (14)$$

The model can be formulated in a matrix structure.

$$y = X\beta + \varepsilon \quad (15)$$

In the above equation, y represents the dependent variable, X is a matrix containing the independent variables, β is the coefficients vector, and ε represents the error terms. The method for estimating the coefficients β is Ordinary Least Squares, defined as:

$$RSS = \sum_{i=1}^n (y_i - y_p)^2 \quad (16)$$

y_p is the predicted value from the regression model. Linear regression aims to determine the line of best fit that minimizes the sum of the squared deviations between the actual data points and the values anticipated by the model. The coefficients and the value of R^2 (coefficient of determination) are presented in Table 2.

Table 2: Calculated coefficients and R^2 .

| a | b | c | d | R^2 |
|------|------|-----|------|-------|
| 0.42 | 0.32 | 0.7 | 0.95 | 0.92 |

3 Results and Discussion

The Nusselt numbers for nanofluids were determined using the equations presented in the data analysis section. Figure 4 illustrates experimental Nusselt numbers for various Reynolds numbers and nanoparticle concentrations. The graph shows a noticeable increase in the Nusselt number with higher Reynolds numbers or volumetric concentrations. For the 1% nanofluid at a Reynolds number of 1238, the Nusselt number is enhanced by around 40% compared to pure water. At a Reynolds number of 4218, the Nusselt number rises by nearly 27% compared to pure water at

the same Reynolds number. These findings show that the heat transfer characteristics of the nanofluid is better at lower Reynolds numbers.

The Correlations developed by Pak et al. (Pak and Cho, 1998) and Duangthongsuk et al. (Duangthongsuk and Wongwises, 2009) have R^2 of 0.70 and 0.75 for the same concentration and Reynolds number. Figure 5 compares these correlations with experimental data for the 1% nanofluid.

Figure 6 demonstrates the comparison between the predicted Nusselt numbers from developed correlation and experimental Nusselt numbers. It shows that correlation can predict Nusselt number accurately. The R^2 value for the developed correlation in the presented research is 0.92.

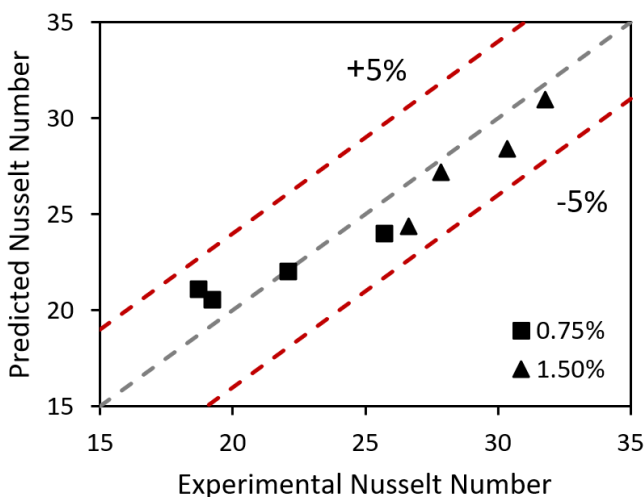


Figure 6: Predicted Nusselt numbers versus experimental Nusselt numbers.

4 Conclusions

This study examines the heat transfer characteristics of a combined nanofluid consisting of suspended Al_2O_3 and TiO_2 dispersed in water. An experimental investigation was conducted using a pressurized test facility with a test section that produces a non-uniform heat flux. The experiment examined the influence of nanoparticle volume fraction and Reynolds numbers on the Nusselt number. The results show that with higher Reynolds numbers and nanofluid concentrations, the Nusselt number improves significantly. The effect of adding nanofluid for heat transfer improvement is much greater at lower Reynolds numbers than at higher Reynolds numbers.

Based on the conducted experiment, a novel correlation for predicting Nusselt numbers was developed using multiple linear regression, which showed an accurate performance with an R^2 value of 0.92. The comparison of the experimental Nusselt number with other correlations and the developed correlation indicates that the new correlation performs better.

Conflict of Interest

The authors declare no potential conflict of interest regarding the publication of this work.

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