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Objective functions performance in the multi-objective fuel management optimization

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HIGHLIGHTS

- The effect of the selection of the cost function has been analyzed.
- The spectrum of randomly generated loading patterns versus multiplication and power peaking factors is calculated.
- It is shown that the rate of convergence is a dependent parameter to the mathematical formulation.

ABSTRACT

Selecting a genuine objective function in the fuel management optimization (FMO) of newly developed reactors is fundamentally important. The FMO problem becomes harder when a multi-objective fitness (cost) function (MOCF) is in use. Usually, when undertaking a MOCF fuel management optimization problem, it is transformed into the summation of objective functions, which are related to weighting factors. Different parameters can be chosen as the main fitness function in an optimization problem. In the case of a nuclear reactor, the cycle length, the multiplication factor and power peaking factor are the most significant. The value of the weighting factors and/or the method with which the cost function has been formulated may affect the final result of optimization. In this paper, the effect of the selection of the cost function has been analyzed in order to reach an optimum in core fuel management of a typical pressurized water reactor, PWR. It is understood from the results that finding a loading pattern that results in a better power peaking factor (lower PPF) is stricter than that of a longer cycle length. Indeed, the obtained loading pattern strongly depends on the selected fitness function. Finally, the flattening function is proposed instead of minimizing the PPF to attain better loading patterns.

KEYWORDS

Fuel management
Fitness function
PSO
SA
Loading patterns

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1 Introduction

Minimizing the cost of any in-core fuel management optimization is critical. In the last four decades, considerable work has been completed employing various optimization techniques in determining the core-loading pattern that minimizes the fuel cost.

The normal loading patterns (LPs) try to reach the flattest possible spatial power distribution during the cycle. However, this category of LP can lead to higher leakage and may lead to a shorter cycle length. Moreover, the number of possible combinations for the fuel assembly (FA) loading in the core is very large. Thus the fuel management optimization has always been deemed one of the most ambitious combinatorial optimization problems in the nuclear industry in both viewpoints mentioned above.

To attain that, many optimization schemes have been developed over the decades. Chen and Mingle (Chen et al., 1977) used linear programming to tackle the fuel management optimization (FMO) problem. Tahara et al. (Tahara et al., 1991) developed a system based on artificial intelligence techniques. Haibach and Feltus (Haibach and Feltus, 1997b), Haibach and Feltus (Haibach and Feltus, 1997a), Francois and López (François and Lopez, 1999), Francois et al. (François et al., 2013), Ortiz and Requena (Ortiz and Requena, 2004) adapted genetic algorithms (GAs) to solve the combinatorial optimization problem. Mahlers (Mahlers, 1994) and Šmuc et al. (Šmuc et al., 1994) used simulated annealing (SA) with linear programming and an adaptive generator of solutions, respectively. Tabu search techniques for solving the fuel loading problem is exercised (Castillo et al., 2004). Moura et al. (de Moura Meneses et al., 2009) employed an algorithm based on particle swarm optimization. Khoshahval

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et al. (Khoshahval et al., 2010) derived a continuous version of particle swarm optimization. DaSilva and Schirru (da Silva and Schirru, 2011) applied an evolutionary algorithm inspired on quantum mechanics. All the optimization techniques inspect for the reload pattern which maximizes/minimizes the Objective Function.

Few studies have been reported in analyzing the effect of the objective function selection in the FMO problem. Here, we focus on the selection of an objective function. To show the effects of selection of objective functions on the optimization of PWR's loading pattern, a hybrid particle swarm optimization along with simulated annealing is developed and applied successfully.

2 Development of a PSO-SA hybrid metaheuristic optimization method

Metaheuristic particle swarm optimization (PSO) is a popular and nature-inspired optimization framework. PSO as a stochastic approach is suited for solving continuous and discrete optimization problems. PSO was first suggested by Kennedy and Eberhart (Kennedy and Eberhart, 1995), which was inspired by social behaviors of bird flockings and fish schoolings. In PSO, randomly generated particles (solutions) spread in the design space to find the optimum solution. Each particle moves in the search space with a velocity according to its own previous best solution (*Pbest*) and its global previous best solution (*Gbest*). *Pbest* and *Gbest* give the particles their specific characteristics. A better particle adapted to the problem has a higher chance of surviving. The position of a particle represents a candidate solution to treat the optimization considered problem. The following equations are used to iteratively modify the particle velocities and positions at each time step, i.e.:

$$v_{id}^{t+1} = w^t v_{id}^t + c_1 r_1^t (Pbest_{id} - x_{id}^t) + c_2 r_2^t (Gbest_{id} - x_{id}^t) \tag{1}$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \tag{2}$$

where $i = [1, 2, ..., n]$, $d = [1, 2, ..., m]$, n is number of particles in a group, and m is elements of particle vectors. Also, v_{id}^t is the velocity of the particle at time step t, x_{id}^t is the position at time step t, c_1 and c_2 are acceleration constants, r_1 and r_2 are random number between 0 and 1, w^t is the inertia weight at time step t, $Pbest_{id}$ is the previous best position of the particle at time step t, $Gbest_{id}$ is the best position among all particles at time step t, and t is the current iteration.

One of the hindrances of this method is the parameter dependency of the method. As in any optimization algorithm, the PSO parameters are very important since they have a significant impact on optimization results. The number of particles and the maximum iteration number is selected between 20 and 100, respectively, as suggested by Khoshahval et al. (Khoshahval et al., 2010). The other drawback of PSO is its premature convergence, especially when PSO is processing complex and combinatorial problems. Indeed, the precept causing the mentioned issue is that for global best PSO, particles converge to a single point, which is on the line between the *Gbest* and the personal *Pbest* positions (Premalatha and Natarajan, 2010).

Simulated Annealing (SA) is the process of locating a favorable approximation to the global minimum of a given function in a large search space. The applied algorithm incorporates SA into PSO when the *Gbest* particle stagnates. This can be done by changing the acceptance criteria of a particle to probability $\exp(-\Delta E/KT)$, even though the current position is poor (K is Boltzmann constant and Δ is a change in the objective function value).

The developed PSO-SA method can be summarized as below:

- Step 1: Specification of the parameters for PSO.
- Step 2: Initialization of particle positions and velocities.
- Step 3: Decode particles.
- Step 4: Run core calculation code.
- Step 5: Evaluate the fitness function values for each of the particles.
- Step 6: Update the position and velocity according to Eqs. (1) and (2).
- Step 7: Comparing current fitness value of each particle with the fitness value of its *Pbest*. Then updating the *Pbest*. If the current *Pbest* value is better than the previous one.
- Step 8: Finding the best particle (best fitness value) of the current swarm. If the fitness value is better than the value of *Gbest*, then update *Gbest* and its fitness value.
- Step 9: Update the particle positions and velocities.
- Step 10: Check the *Gbest* stagnation. If *Gbest* position is not changed over a period of time, then find the new position using new temperature (*nGbest*).
- Step 11: Accept the new position as *nGbest* position with probability $\exp(-\Delta E/T)$ even though the current position is worse.
- Step 12: Update temperature (reduce Temperature).
- Step 13: Check the stopping criteria. Terminate if the condition is met (user-defined stopping criterion such as reaching to maximum iteration number).
- Step 14: Go to step 3.
- Step 15: Printing the results of the optimization.

To make it more clearly understood, Fig. 1 illustrates the proposed procedure of the PSO-SA.

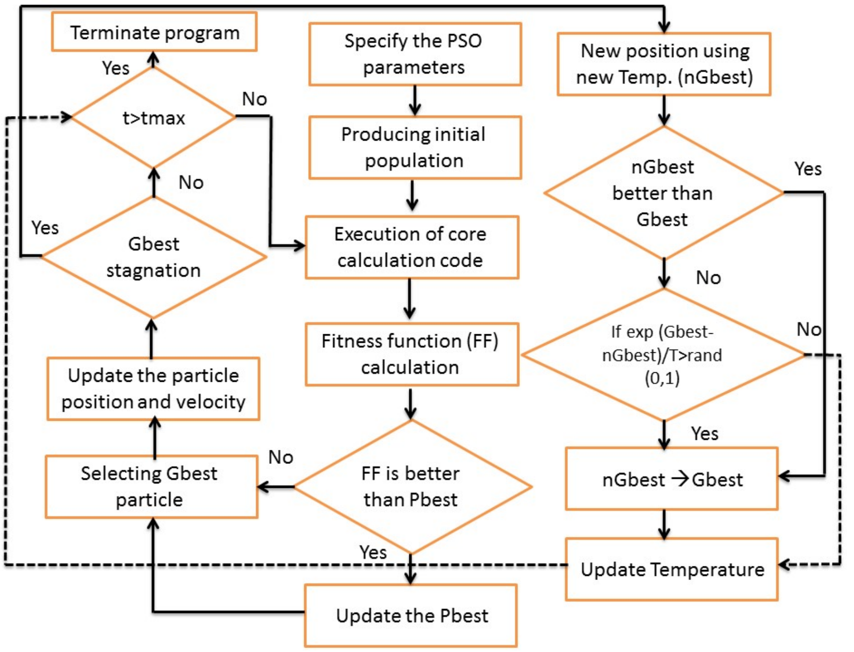

Figure 1: The developed PSO-SA method for in core fuel management optimization.

3 Numerical calculations

In the present work, the core calculation is carried out by FEMPT code (Khoshahval et al., 2014). FEMPT is a core calculation code which has been developed to solve multigroup transport equations in 2-D. In addition, a FORTRAN language program is provided to make the linkage between different parts (See Fig. 1). Also, for the generation of macroscopic cross sections and constants, the WIMSD5 code has been used (Halsall, 1982). It is worth mentioning that the current work focuses on the issue of the objective function and the calculations are done for the beginning of the cycle of the reactor.

4 Parameter setting

Parameter tuning is not an easy task; however, it deeply depends on which specific optimization method will be used and on the fitness function to be optimized. Sometimes another optimization method is needed to enhance the parameters of the main applied optimization method. In this paper, the parameters of PSO-SA are set based on our experience as well as other research results reported in the literature. The acceleration constants are set to 2.0, an initial number of particle population is set to 20, and the maximum iteration number is set to 100.

In the loading pattern optimization problems, the previous investigations show that there is a very diverse set of objective functions used in the optimization problems. However, finding appropriate objective functions is not easy. The common or main parameters in most of the applied objective functions in the in-core fuel management optimization include minimization of power peaking factors and the maximization of the cycle length. Most of the studies have been focused on the increasing of the reactor core's cycle length and improve the safety margin for preventing accident (Stamford and Azapagic, 2012). The selection of objective function can change the search space. As stated, minimization of power peaking factor and maximizing cycle length (increasing the multiplication factor) are the most common objective functions used in the heuristic optimization methods. It is difficult to accomplish both of these objective functions simultaneously. For the first step, we attempt to analyze the search space of a typical PWR from the viewpoint of the two mentioned main parameters. Here, our test case is a PWR reactor with 193 fuel assemblies. Each fuel assembly contains 236 fuel rods and 20 guiding channels for burnable poisons or control rods. Table 1 shows the design data of the KWU reactor. The full core model of the KWU reactor at the beginning of the cycle (BOC) has been shown in Fig. 2.

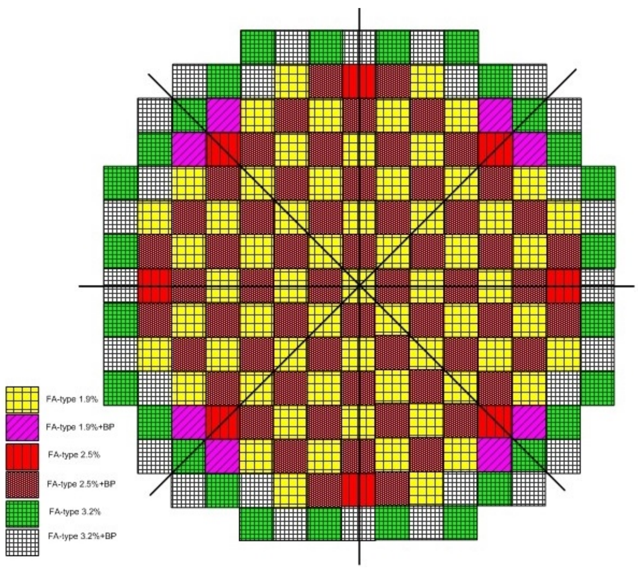

Figure 2: A typical PWR reactor core cross vie.

Table 1: Design data for the KWU Nuclear Power Plant (Bushehr old design).

Parameter	Value
Reactor nominal thermal power (MW)	~3300
Number of fuel assemblies	193
Core active height (cm)	390.0
Equivalent core diameter (cm)	360.5
Enrichment distribution in 1st loading (1.9/2.5/3.2) weight (%)	69/68/56
Average power density (kw/l)	93
Fuel material	UO_2
Clad material	Zr
Moderator	Water
Fuel pellet diameter (mm)	9.11
Clad outer diameter (mm)	10.75
Fuel rods pitch (mm)	14.35
Clad thickness (mm)	0.72
Pressure in vessel (bar)	158
Inlet temperature ©	291.3
Outlet temperature ©	326.1
Fuel assembly grid	16×16
Number of fuel rods per assembly	236
Number of control rod guide thimbles per assembly	20
Guide thimble (material/outer dia. (mm)/wall thickness (mm))	S.S./13.72/0.42

Six thousands of randomly generated particles (arrangements) are produced and their corresponding effective multiplication factors and power peaking factor are evaluated. Figure 3 shows the spectrum of the 6000 randomly generated particles.

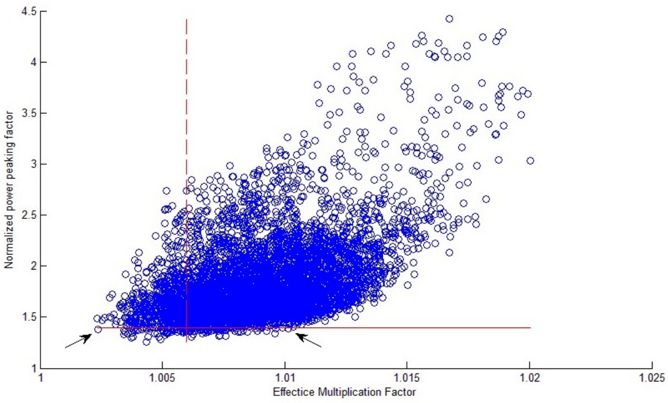

Figure 3: Multiplication factor versus power peaking factor for six thousands of randomly generated loading patterns.

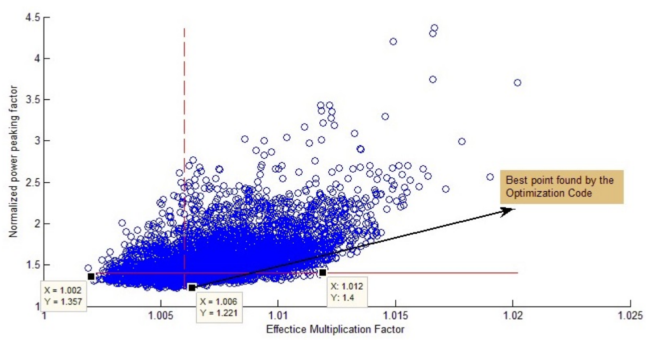

Figure 4: Multiplication factor versus power peaking factor for optimized six thousands of loading patterns shown in Fig. 2.

As can be seen from Fig. 3, the spectrum of generated LPs is not uniform although the random number generated was uniform. The lowest value of the effective multiplication factor was 1.0023, and the highest was 1.0200. The lowest value of power peaking factor was 1.2570, and the highest was 4.425. Among the 6000 LPs, just 124 LPs had the acceptable value (from a safety point of view, the maximum peaking factor is considered to be less than 1.40). In this range, the multiplication factor alters from 1.0024 to 1.0101, while the power peaking factor changes from 1.2570 to 1.399. As can be seen from Fig. 3, fluctuations of the effective multiplication factors are less than fluctuations in the power peaking factors, so one can put more emphasis upon the maximization of the multiplication factor. It should be noted that for this test case, the burnup calculation is not included, and the effective multiplication factor, K_{eff}, is for the beginning of the cycle. Therefore, one can say that although the number of possible locations for fuel assemblies is huge, the number of acceptable LPs are confined. This is the main reason that we use the penalty function in the objective functions. This penalty function causes bypass calculations on these varieties of LPs and decreases the time needed to reach the best answer (less convergence time).

Now, one can put the random generated LPs as an initial population in any optimization methods (here PSO-SA). The resulted optimized pattern is depicted in Fig. 4. As can be seen from this figure, the number of LPs under the horizontal line is obviously increased. More precisely, among 6000 arrangements, 1343 arrangements have now power peaking factors less than the considered criteria (1.40). This means that the number of acceptable LPs is increased by a factor of 11. The lowest value of the optimized patterns multiplication factor is 1.0019 and the highest is 1.0202. In addition, the lowest value for power peaking factor is 1.221 and the highest one is 4.367. Among the 1343 arrangements, the Keff varies between 1.0020 and 1.0118, while the power peaking factor changes between 1.221 and 1.399. The best arrangement (from view point of the two considered parameters) is shown also in Fig. 4 by an arrow. The best-found LP gives 1.0063 and 1.221 for Keff and power peaking factor respectively. However, it should be noted here, that in practice for optimization methods, we do not need to start with 6000 particles (chromosome in the genetic algorithm). For example, for the number of particles in the PSO method, 20 or 50 particles is enough.

4.1 Power peaking factor minimization

In order to have an effective search process in the presence of a constraint, the fitness function needs to be carefully defined. Instead of using a simple objective function for minimizing the power peaking factor, the multi-objective cost function to be discussed here is considered, as below, to minimize the power peaking factor via flattening of the normalized power.

$$FF = \sum_{i=1}^{N} (np_i - 1)^2 + F_p \tag{3}$$

$$F_p = \begin{cases} \dfrac{ppf}{ppf_{max}} \left(\sum_{i=1}^{N} (np_i - 1)^2 \right) & \text{if } ppf > ppf_{max} \\ 0 & \text{if } ppf < ppf_{max} \end{cases} \tag{4}$$

According to the stochastic behavior of heuristic optimization method, after each run, the final answer can be different. During optimization, using Eqs. (3) and (4), we found two optimized LPs with special features (named $opt-1$ and $opt-2$). Table 2 shows the optimization results of these two optimized patterns.

As can be seen from Table 2, the fitness value of the $opt-2$ is less than that of $opt-1$. This means that $opt-2$ is more flattened than $opt-1$, but the maximum power peaking factor for $opt-2$ is higher than $opt-1$. In other words, selecting only a single objective of lowering power peaking factor cannot guarantee the flatness of the optimized pattern and vice versa. However, both optimized patterns have the power peaking factor less than the criteria. Thus, by adjusting the penalty function (Eq. (4)), one can reach a pattern with both the highest amount of flatness and at the same time lowest power peaking factor. Indeed, the fitness function is augmented intentionally by adding nonnegative penalty term F_p, penalizing constraint violations. Arrangements of $opt-1$ and $opt-2$ and their relative power distribution are illustrated in Fig. 5 and Fig. 6, respectively.

Table 2: Power peaking and effective multiplication factor for fitness function FF.

Parameter	$opt-1$	$opt-2$	ref
K_{eff}	1.007033	1.00389	1.004258
ppf	1.31	1.33	1.36
FF	1.087424	1.021767	1.677525

Table 3: Fit_2 results, PSO-SA.

	Reference (objective) value (Khoshahval et al., 2014)	Best Result$-Fit_2$
K_{eff}	1.00485	1.00488
PFF_{max}	1.273	1.272
Fit_2	1.22334	0.75627

Table 4: Fit_3 results, PSO-SA.

	Reference (objective) value (Khoshahval et al., 2014)	Best Result$-Fit_2$
K_{eff}	1.00485	1.00650
PFF_{max}	1.273	1.245
Fit_2	1.46381	0.91897

4.1.1 Increasing effective multiplication factor at the beginning of the cycle

The most common method to increase the cycle length is to maximize the multiplication factor (Driscoll et al., 1990; Jeong et al., 2018). Here, two different formulas deemed for maximization of cycle length, while keeping the power peaking factor as low as possible, Eq. (5) and Eq. (6), have the same calculation cost but just a different formulation:

$$Fit\_2 = w_1 FF - w_2 K_{eff} \tag{5}$$

$$Fit\_3 = FF / K_{eff} \tag{6}$$

According to Table 3 the used objective function has the ability and reliability for finding an optimum pattern.

The same procedure is done for the Fit_3 (Eq. (6)). Thus, we changed the objective function but the developed program still can find the good answers in comparison to reference value. Moreover, the comparison between Table 3 and Table 4 show that the Fit_3 function outperforms Fit_2 (higher multiplication factor and lower power peaking factor). However, it should be noted that standards deviations of results attained from Fit_2 are better than Fit_3.

Moreover, in order to show the effect of using the weighting factors and to demonstrate their importance, another fitness function is accomplished by Fit_4 (See Eq. (7)).

$$\begin{aligned} Fit\_4 &= w_f \times F_1 - S_{12} \times w_k \times F_2 \\ F_1 &= ((PPF - PPF_{ref})/PPF_{ref}) \\ F_2 &= ((KEFF - KEFF_{ref})/KEFF_{ref}) \end{aligned} \tag{7}$$

$$S_{12} = \left| \frac{F_1}{F_2} \right| \tag{8}$$

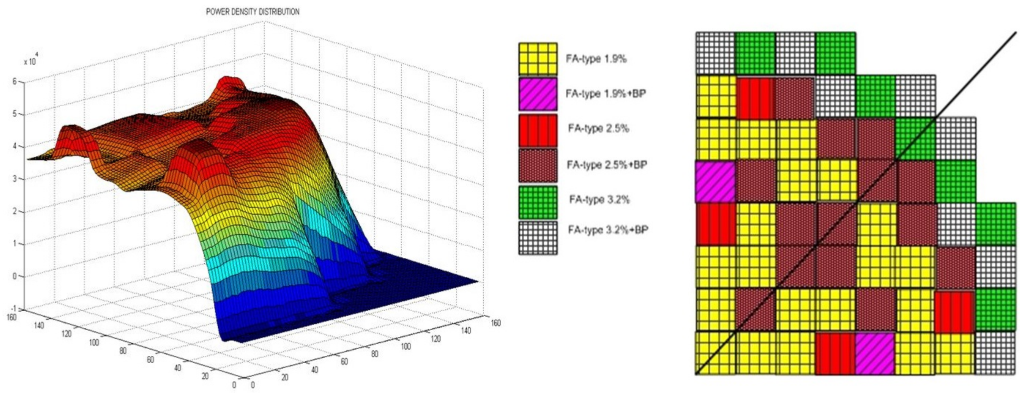

Figure 5: Power distribution (left picture) and loading pattern (right picture) of $opt-1$.

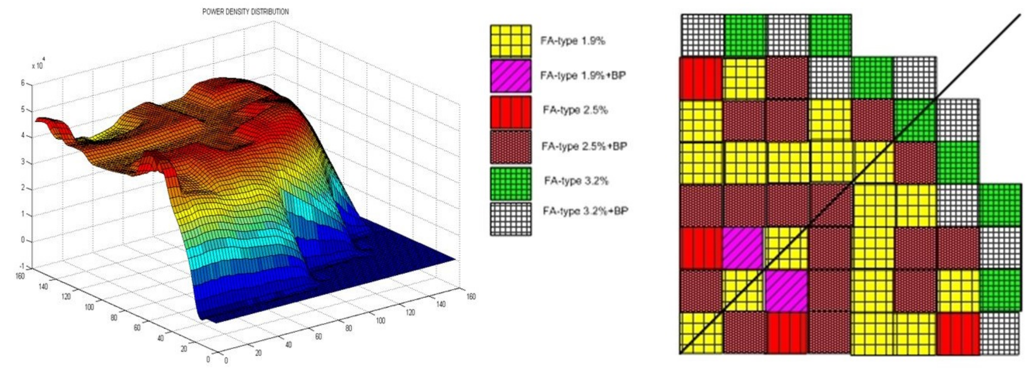

Figure 6: Power distribution (left picture) and loading pattern (right picture) of $opt-2$.

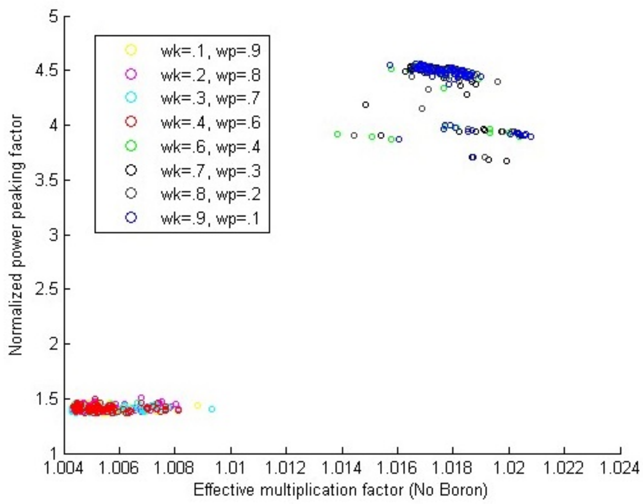

Figure 7: Power peaking factor versus effective multiplication factor for differenct set of weighting factors.

Figure 7 shows the normalized power peaking factor versus the effective multiplication factor. For each set of the weighting factors, the developed software is executed 100 times. As can be seen from Fig. 7 data in the search space have been divided into two parts. Thus, one can adjust the weighting factors based on the considered design parameters for the reactor core and to obtain suitable fuel utilization. Indeed, based on the main considered core function, a greater emphasis may be placed upon one weighting factor or another. In other words, the quality of the outputs of the applied optimization algorithm strongly depends on the fitness function and weighting factors selection.

5 Conclusion

In this work, the main goal was to explore PSO-SA performance by changing the fitness functions. In this investigation, the spectrum of randomly generated LPs versus multiplication factor and power peaking factor is calculated and plotted. After applying the optimization method, the numbers of acceptable LPs are increased by a factor of 11. We also showed how the used objective function in optimization methods affects the final result. Moreover, it is shown that the rate of convergence is a dependent parameter to the mathematical formulation of the objection function. In addition, we understood that Fit_3 function outperforms Fit_2 (higher multiplication factor and lower power peaking factor).

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