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Investigation of the degeneracy effect on the inverse bremsstrahlung absorption using the average-atom model in inertial confinement fusion

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HIGHLIGHTS

- In low photon energy, the electrons of plasma absorb the photon energy and transfer to upper levels of energy.
- In high photon energy, the absorption coefficient is smaller than the refraction coefficient.
- The plasma is optically thin in high photon energy, and the photon escapes from the plasma.
- In low photon energy, plasma is optically thick, therefore the photon is absorbed by electrons of the plasma.
- The degeneracy has caused a reduction in opacity and the inverse bremsstrahlung absorption.

ABSTRACT

The inverse bremsstrahlung absorption is one of the most absorption processes in inertial confinement fusion plasma. In the present work, some important optical properties of plasma, such as index of refraction, absorption coefficient, electrical conductivity, and the electrical dielectric function in degenerate conditions are presented from an average atom model's point of view. To include quantum diffraction effects, the Coulomb potential is replaced by an effective quantum potential for a screened electron-ion interaction, named Klebg potential. Therefore, the electrical conductivity contribution of the free-free electrons absorption that is inverse of the bremsstrahlung absorption, for transition from state i to state j is obtained using the wave function of this potential. By this method, the effect of degenerate plasma on the inverse bremsstrahlung power is calculated using the free-free absorption coefficient. Finally, the obtained results of the free-free optical opacity of plasma and the inverse bremsstrahlung absorption power from classical and degenerate plasma are compared.

KEYWORDS

Degenerate plasma
Inverse bremsstrahlung losses
Index of refraction
Conductivity
Dielectric function

HISTORY

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1 Introduction

Many radiation and absorption phenomena is occurred in inertial confinement fusion, such as bremsstrahlung radiation, Campton scattering, inverse Compton scattering, and inverse bremsstrahlung absorption (Eliezer et al., 1998). In fact, optical parameters of plasma are determined via various modes of interaction between radiation and plasma through absorption (bound-bound, bound-free and free-free) and scattering (Arkhipov et al., 2020). Radiation processes in plasma involve electrons and photons with positive energy levels ($E > 0$) and negative energy levels ($E < 0$) for electrons. In the present work, we study the inverse bremsstrahlung absorption in degenerate plasma.

If electron temperature is lower than the Fermi temperature, the degeneracy effects is occurred in fusion plasma. In degenerate plasma, in low temperature and high density, the quantum effects play an important role. Because of the exclusion principle, some radiation transitions is forbidden which reduces the total ion-electron collision rate from the classical prediction. Owing that the inverse bremsstrahlung absorption heating results from electron-ion collisions, in degenerate plasma, this phenomena is also reduced (Son and Fisch, 2006).

Inverse bremsstrahlung absorption occurred by free-free electron transition from a positive energy level to a higher positive energy level of absorbing a photon. Moll *et al.* calculated the inverse bremsstrahlung absorption such as an important process in the laser-matter interaction, involves two different kinds of interactions: the interaction of the electrons with the external laser field, and the electron-ion interaction in a classical plasma in inertial

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confinement fusion (Moll et al., 2012).

In degenerate plasma, the final state might be populated with the other electrons and some transition is limited by the Pauli Exclusion Principle (Bailey et al., 2009). The purpose of this article is to study some optical properties theoretically using the average ion model in degenerate plasma. This model is similar to a model previously used by Johnson *et al.* (Johnson et al., 2006). This model is a quantum mechanical version of the Tomas-Fermi model of plasma and it has been applied to calculate the frequency-dependent conductivity $\sigma(\omega)$ of the plasma using the Kubo-Greenwood (KG) formula. They determined some optical properties of plasmas, including dielectric constants, indices of refraction, and absorption coefficients, from an average-atom point of view.

By this method, the real and imaginary parts of dielectric function will be obtained with the Kramers-Kronig dispersion relations (Johnson et al., 2006). In the present work, the plasma opacity for free-free electrons as function of plasma temperature, density and radiation frequency is discussed by calculating the absorption coefficient. Finally, the total inverse bremsstrahlung absorption power is calculated in degenerate plasma in inertial confinement fusion.

2 Physical Model

We consider the scattering of an electron on an atomic ion in hydrodynamic expansion of the plasma in inertial confinement fusion. In the inverse bremsstrahlung process, while one electron passing through the field of an ion to absorb radiation, temperature of the electron is raised. Therefore, in dense plasmas, the inverse bremsstrahlung heating plays an important role in increasing electron temperature. In order to derive the inverse bremsstrahlung absorption power, we start from the Schrödinger equation for electrons and ions in plasma (Moll et al., 2012):

$$\left[\frac{P^2}{2m_{ei}} + V(r)\right]\psi(r) = \varepsilon\psi(r) \tag{1}$$

where $V(r)$, ε, and m_{ei} are the ion-electron interaction potential, energy, and reduced mass of ion and electron, respectively. In a classical plasma, there is the Coulomb potential between ion and electron which changes to $1/r$. However, in degenerate conditions, the potential is different. To include quantum diffraction effects for a two-component electron-ion plasma, one can replace the Coulomb potential by an effective quantum potential for a screened electron-ion interaction which was derived by Kelbg. Accordingly, the reaction potential is (Ebeling et al., 2006)

$$V(r) = \frac{1}{r}\{1 - \exp(-\frac{r^2}{\Lambda_{ei}})\} + \frac{\sqrt{\pi}}{\Lambda_{ei}}\{1 - \mathrm{erf}(\frac{r}{\Lambda_{ei}})\} \tag{2}$$

where the standard Error function is:

$$\mathrm{erf}(\frac{r}{\Lambda_{ei}}) = \frac{2}{\sqrt{\pi}}\int_0^{\frac{r}{\Lambda_{ei}}} \exp(-t^2)\,\mathrm{d}t \tag{3}$$

In this potential, the electron and ion do not see each other as point particles, but as charge clouds with an effective diameter (the thermal-wavelength):

$$\Lambda_{ei} = \frac{h}{\sqrt{2m_{ei}KT}} \tag{4}$$

where KT is the plasma temperature. While in a classical plasma, the distribution function is Maxwell-Boltzmann, in a degenerate plasma, the electrons' distribution function is Fermi-Dirac. As a result, the wave function for describing each particle, considering the Kelbg potential in the Schrödinger equation, is presented by the Gaussian wave pocket as (Zwicknagel and Pschiwul, 2006):

$$\begin{aligned}\psi_i(r) =&(\frac{3}{2\pi\gamma^2})^{3/4}\times \\ &\exp\{-[\frac{3}{4\gamma^2} - \frac{ip_\gamma}{2\hbar\gamma}](r-R_i)^2 + \frac{i}{\hbar}P_i(r-R_i)\}\end{aligned} \tag{5}$$

where R_i and P_i are the particle coordinate and momentum in i state. Also, γ and p_γ are the width of Gaussian wave pocket and its momentum. The electrical conductivity is one of the important properties of plasma for different types of emissions and absorptions (free-free, free-bound, bound-bound). The electrical conductivity contribution of the free-free electrons absorption (inverse bremsstrahlung) for transition from state i to state j is obtained using the KG formula (Johnson et al., 2006):

$$\begin{aligned}\sigma(\omega) = \frac{8\pi e^2}{3\Omega\omega}\sum_{l_i}\int_0^\infty \mathrm{d}\varepsilon_i\,(f_i - f_j) \\ \times\left[|\langle\psi_i(r)|V(r)|\psi_j(r)\rangle|^2\right]^2\end{aligned} \tag{6}$$

where $\varepsilon_j = \varepsilon_i + \omega$, and f is the Fermi-Dirac distribution function in the i and j states, and photon energy, respectively. Now, the complex dielectric function is defined as:

$$\varepsilon_r(\omega) = 1 + i\frac{4\pi}{\omega}\sigma(\omega) \tag{7}$$

The real and imaginary parts of conductivity are obtained from the KG formula and the Kramers-Kroning dispersion relations, respectively:

$$\mathrm{Im}\,\sigma(\omega) = -\frac{2\pi}{\omega}\int_0^\infty \mathrm{d}\omega_0\,\frac{\mathrm{Re}\,\sigma(\omega_0)}{\omega_0^2 - \omega^2} \tag{8}$$

Therefore, the real and imaginary parts of dielectric function are determined by:

$$\mathrm{Re}\,\varepsilon_r(\omega) = 1 - 4\pi\frac{\mathrm{Im}\,\sigma(\omega)}{\omega} \tag{9}$$

and

$$\mathrm{Im}\,\varepsilon_r(\omega) = 4\pi\frac{\mathrm{Re}\,\sigma(\omega)}{\omega} \tag{10}$$

In relating the complex dielectric function and complex conductivity to the optical properties of plasma such as absorption and refraction, it is convenient to introduce a complex index of refraction which is related to the dielectric function as:

$$\sqrt{\varepsilon_r} = \eta + i\kappa \tag{11}$$

where η (index of refraction) and κ (index of absorption) are collectively called the optical constants and can be obtained as (Dresselhaus et al., 2018):

$$\eta(\omega) = \sqrt{\frac{\sqrt{[\mathrm{Re}\varepsilon_r(\omega)]^2 + [\mathrm{Im}\varepsilon_r(\omega)]^2} + \mathrm{Re}\varepsilon_r(\omega)}{2}} \tag{12}$$

and

$$\kappa(\omega) = \sqrt{\frac{\sqrt{[\mathrm{Re}\varepsilon_r(\omega)]^2 + [\mathrm{Im}\varepsilon_r(\omega)]^2} - \mathrm{Re}\varepsilon_r(\omega)}{2}} \tag{13}$$

By considering the relation between refraction index and wave number as:

$$\eta + i\kappa = \frac{c}{\omega}(\kappa + i\acute{\kappa}) \tag{14}$$

in which κ and $\acute{\kappa}$ are the wave number and the absorption coefficient, the opacity of plasma for free-free state is calculated by (Atzeni and Meyer-ter Vehn, 2004):

$$\kappa_\nu^{ff} = \acute{\kappa}\,\frac{\sqrt{1 - \omega_p^2/\omega_L^2}}{\rho(1 - \exp(-\omega/KT))'} \tag{15}$$

In this equation, ρ, ω_p and ω_L are the density, the plasma, and laser frequency depending on electron density and wavelength, respectively. The ultra-intense Pico second laser pulse ($\sim 10^{20}$ W.cm^{-2}) is penetrated in plasma for ignition (Mahdavi and Kaleji, 2014). The inverse bremsstrahlung absorption can occur with multiphoton terms for a wide range of frequencies. Therefore, with the integration of free-free opacity on the all frequencies, the inverse bremsstrahlung absorption is obtained:

$$P_{IBR} = 4\pi \int_0^\infty \mathrm{d}\nu\, \kappa_\nu^{ff} \tag{16}$$

The interaction between plasma and radiation is dependent on the plasma properties such as temperature, density, and the frequency of radiation. In degenerate plasma, the electron temperature should be less than the Fermi temperature, that depends on the density of electrons ($KT_F = (\hbar^2/2m_e)(3\pi^2 n_e)^{2/3}$). In this study, the calculations are considered for the electron temperature, $T_e = 3$ keV which is smaller than the Fermi temperature, $T_F = 7.1$ keV for the electron density, $n_e = 10^{27}$ cm^{-3} (Mahdavi et al., 2010).

3 Results and Discussion

In Fig. 1, the electron conductivity is plotted as a function of the incident photon energy in conditions of degenerate plasma, which is a decreasing function of the photon energy. In low photon energy, the electrons of plasma absorb the photon energy and transfer to upper levels of energy. In this condition, the electrons' velocity will be increased. Therefore, the electrical conductivity will be increased in low photon energy.

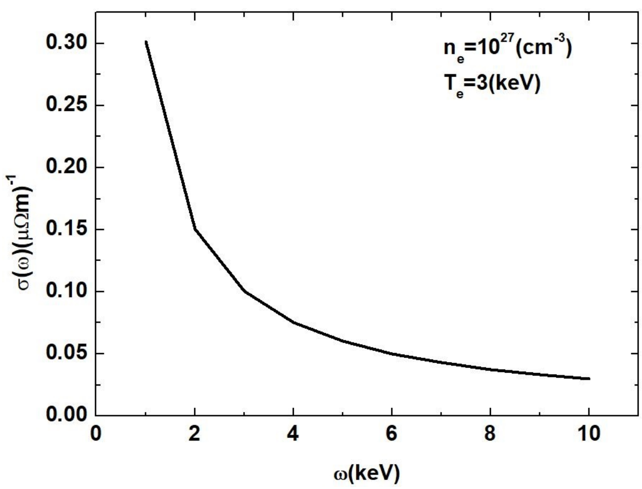

Figure 1: The electronic conductivity versus incident photon energy.

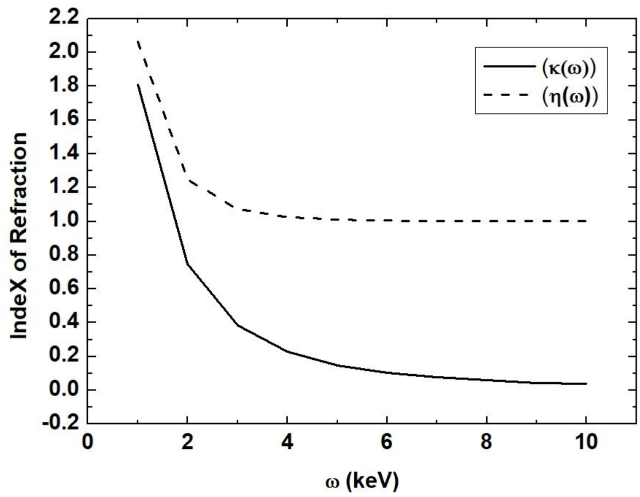

Figure 2: Real and imaginary parts of the index of refraction as function of photon energy.

The real and imaginary parts of the refraction index are shown in Fig. 2, as the function of photon energy. The real part of the refraction index is identified as a refraction contribution and the imaginary part of the refraction index accounts for attenuation of the radiation in plasma. In the high photon energy, the absorption coefficient is smaller than the refraction coefficient. The plasma is optically thin in high photon energy, and the photon escapes from the plasma. Hence, the refraction process is larger than the absorption process.

Moreover, the inverse bremsstrahlung free-free opacity in degenerate and classical plasma are compared in Fig. 3. The dashed line shows the free-free opacity of plasma in a classical plasma with considering the Maxwell-Boltzmann distribution function. The classical wave function in plasma and the solid line shows the free-free opacity of plasma in a degenerate plasma with considering the Fermi-Dirac distribution function, where Eq. (4) is used for wave function. While the classical plasma includes low density and relatively high electron and ion temperature, the degenerate plasma have a high number density and relatively low temperature (Johnson et al., 2006). In Fig. 3, it is shown that the degeneracy decreases the opacity. This behavior is a consequence of the prevention of transition due to the Pauli exclusive principle in degenerate plasma. In low photon energy, plasma is optically thick, and therefore, the photon is absorbed by electrons of the plasma. Accordingly, the free-free opacity of the high electron temperature is larger than the low electron temperature.

In Fig. 4, the calculation of inverse bremsstrahlung power is compared in degenerate and classical plasma for different densities. The solid line shows the opacity in density 10^{27} cm^{-3}, where there is a degeneracy condition, and the dashed line represents the opacity in classical plasma condition ($n_e \sim 10^{23}$ cm^{-3}). It is shown that the inverse bremsstrahlung for degenerate plasma is lower than classical plasma, but this difference is omitted by increasing the electron temperature. Therefore, the degeneracy has caused the losses process, then the inverse bremsstrahlung absorption will be decreasing in the degenerate condition.

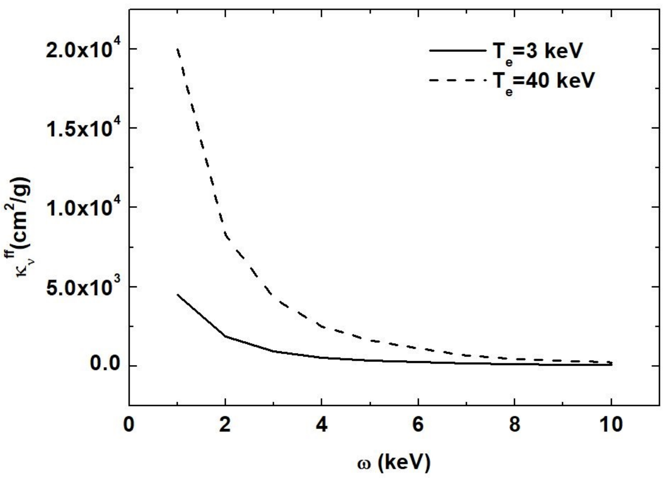

Figure 3: The free-free opacity of plasma versus photon energy for different electron temperatures.

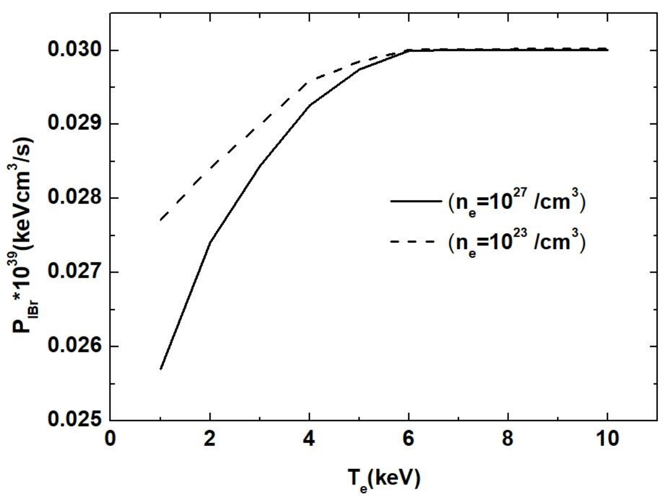

Figure 4: The inverse bremsstrahlung absorption power versus electron temperatures in degenerate (solid line) and classical (dashed line) plasma.

4 Conclusion

Inverse bremsstrahlung absorption is the most efficient absorption mechanism in laser-fusion plasma. We extended this phenomena here to non-ideal contributions. In this work, an average-atom model is used to determine the frequency dependent conductivity by considering the Gaussian wave function and Kelbg potential for plasma particles. Through using the dispersion relations, the properties of plasma such as the index of refraction and the electrical dielectric function are calculated. The coefficient of free-free absorption and the inverse bremsstrahlung power are calculated in classical and degenerate plasma. The results have shown that the degeneracy has caused a reduction in opacity and the inverse bremsstrahlung absorption. In high density ($n_e \sim 10^{23}$ cm^{-3}) and low temperature, when plasma is degenerated as seen in Fig. 4, the inverse bremsstrahlung absorption will be decreased. Therefore, one of loss processes in degenerate plasma decreased and prevents the plasma from pre-heating up.

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